

# Current-driven spin injection from a probe to a ferromagnetic film

S. G. Chigarev, E. M. Epshtein\*, Yu. V. Gulyaev,  
A. I. Panas, V. G. Shofman, P. E. Zilberman

V.A. Kotelnikov Institute of Radio Engineering and Electronics  
of the Russian Academy of Sciences, 141190 Fryazino, Russia

## Abstract

The distribution is calculated of the electron spin polarization under current-driven spin injection from a probe to a ferromagnetic film. It is shown that the main parameters determining difference of the spin polarization from the equilibrium value are the current density and the spin polarization of the probe material, while the relation between the probe diameter and the spin diffusion length influences the result very weakly, to a certain extent. A possibility is shown of reaching inverse population of the spin subbands at distances from the probe boundary comparable with the spin diffusion length.

## 1 Introduction

The current-induced spin injection, i.e., appearing nonequilibrium spin polarization near boundary between two conductors under current flowing through [1], is one of the main spintronic effects together with the effect of spin torque transfer from conduction electrons to lattice [2, 3]. The spin injection manifests itself as breaking the thermal equilibrium between spin subbands.

Nowadays, the spin torque transfer effect has been studied with much more details than the spin injection. However, an attractive problem occurs that relates with reaching high injection level. We mean a possibility of creating inverse population of the spin subbands in a ferromagnet with laser effect in THz and IR ranges ( $10^{12}$ – $10^{14}$  Hz) [4]–[9]. The main obstacle for this idea is necessity of high current density  $j \geq 10^9$  A/cm<sup>2</sup>.

A scheme was proposed for reaching high current density in a microprobe–thin film system [10]. If the film thickness  $h$  is small compared to the probe radius  $R$ , then the current density in the film near the probe is  $R/2h$  times the current density in the film. So a problem appears of calculating spin

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\*E-mail: eme253@ms.ire.rssi.ru

injection in such a system as a function of the system parameters and the current value. Such a problem is considered in the present work.

## 2 The model and main equations

Let us consider a ferromagnetic film of  $h$  thickness which current  $I$  is carried to by means of a cylindrical probe of  $R$  radius. The current in the film is spin-polarized, so that its flowing is accompanied with appearing a nonequilibrium spin polarization near the probe at distances comparable with the spin diffusion length  $l$ .

The spin polarization is defined as

$$P = \frac{n_+ - n_-}{n}, \quad (1)$$

where  $n_+$  and  $n_-$  are the partial densities of the conduction electrons with spin magnetic moments aligned parallel and antiparallel to the crystal lattice magnetization vector, respectively,  $n = n_+ + n_-$  is the total density of the conduction density, which is assumed to be constant. It follows from Eq. (1)

$$n_{\pm} = \frac{n}{2}(1 \pm P). \quad (2)$$

The spin polarization distribution under steady conditions is determined by the spin current continuity equation

$$\nabla \mathbf{J} = -\frac{\hbar n}{2} \frac{P - \bar{P}}{\tau}, \quad (3)$$

where  $\bar{P}$  is the equilibrium spin polarization,  $\tau$  is the relaxation time of the longitudinal (collinear with the lattice magnetization) component of the electron spin polarization,

$$\mathbf{J} = \frac{\hbar}{2e}(\mathbf{j}_+ - \mathbf{j}_-) \quad (4)$$

is the spin current density

$$\mathbf{j}_{\pm} = e\mu_{\pm}n_{\pm}\mathbf{E} - eD_{\pm}\nabla n_{\pm}, \quad (5)$$

are the partial current densities created by the electrons with two opposite spin directions,  $\mathbf{E}$  is the electric field,  $\mu_{\pm}$  and  $D_{\pm}$  are the partial mobilities and the partial diffusion constants, respectively.

The electric field  $\mathbf{E}$  can be expressed via the total current density  $\mathbf{j} = \mathbf{j}_+ + \mathbf{j}_-$ . The spin current density (4) takes the form

$$\mathbf{J} = \frac{\hbar}{2e}\{Q(P)\mathbf{j} - eD(P)\nabla P\}, \quad (6)$$

where

$$Q(P) = \frac{\mu_+ - \mu_- + (\mu_+ + \mu_-)P}{\mu_+ + \mu_- + (\mu_+ - \mu_-)\bar{P}}, \quad (7)$$

$$D(P) = \frac{\mu_+D_- + \mu_-D_+ + (\mu_+D_- - \mu_-D_+)P}{\mu_+ + \mu_- + (\mu_+ - \mu_-)P}. \quad (8)$$

Substitution of Eq. (6) to Eq. (3) gives an equation for the spin polarization  $P$  which is substantially nonlinear as follows from Eqs. (2), (7) and (8). Besides the explicit linear-fractional dependence of  $Q(P)$  and  $D(P)$  coefficients on  $P$ , the partial mobilities and diffusion constants of a degenerate electron gas (metal) depend in general on the Fermi quasilevels of the spin subbands and, subsequently, on the partial densities  $n_{\pm}$ , which, in their turn, are expressed via spin polarization  $P$  (see Eq. (2)). The form of that dependence is determined by many factors, such as the form of the spin subbands (dispersion law), the carrier scattering mechanism, etc.

In many cases, such as the problem of the switching magnetic configuration by spin-polarized current, a linear approximation in the current density  $\mathbf{j}$  and proportional to it nonequilibrium spin polarization  $\Delta P = P - \bar{P}$  appears to be sufficient. In this approximation, the spin polarization  $P$  in  $Q(P)$  and  $D(P)$  coefficients is replaced with its equilibrium value  $\bar{P}$ , so that the coefficients mentioned take constant values  $\bar{Q} \equiv Q(\bar{P})$  and  $\bar{D} \equiv D(\bar{P})$ , and Eq. (3) becomes linear equation, namely,

$$\nabla^2 P - \frac{P - \bar{P}}{\bar{l}^2} = 0, \quad (9)$$

where  $\bar{l} = \sqrt{D\tau}$ .

The linear approximation becomes invalid under high spin injection corresponding to the spin subband inverse population, when  $P < 0$ , so that  $\Delta P < 0$ ,  $|\Delta P| > \bar{P}$  [8]. However, the situation is simplified noticeably if it is supposed that the carriers in both spin subbands have the same mobilities and diffusion constants,  $\mu_- = \mu_+ \equiv \mu$ ,  $D_- = D_+ \equiv D$ . In such a case, we have  $Q(P) = P$ ,  $D(P) = D$ . The substitution of Eq. (6) into Eq. (3) with the electric charge conservation condition  $\nabla \mathbf{j} = 0$  taking into account gives the equation

$$\nabla^2 P - \frac{\mathbf{j} \nabla P}{j_D l} - \frac{P - \bar{P}}{l^2} = 0, \quad (10)$$

where

$$l = \sqrt{D\tau}, \quad j_D = enD/l = enl/\tau. \quad (11)$$

The form of this equation depends on neither the carrier degeneration, nor the dispersion law, nor the scattering mechanism. The solution of such simplified problem, without having any pretension to obtaining quantitative results for particular materials, allows to find a qualitative picture and estimate the orders of magnitude.

### 3 Spin polarization distribution

Under spin injection conditions, the spin polarization differs from its equilibrium value at the distances from the injector comparable with the spin diffusion length  $l$ . If the film lateral size is large in comparison with that length, the current density distribution may be considered as axially symmetrical one irrespective of the geometry of the other electrode closing the electric circuit. It follows from the electric charge conservation condition

$$\nabla \mathbf{j} = \frac{1}{r} \frac{d}{dr}(rj) = 0 \quad (12)$$

that the current density distribution in the film near the probe takes the form

$$j(r) = j(R) \frac{R}{r} = \frac{I}{2\pi\hbar r}, \quad (13)$$

where  $r$  is the distance from the probe axis, the other notations being indicated above.

The substitution of Eq. (13) into Eq. (10) gives the following equation in polar coordinates:

$$\frac{d^2P}{dr^2} + (1 - 2\nu) \frac{1}{r} \frac{dP}{dr} - \frac{P - \bar{P}}{l^2} = 0, \quad (14)$$

where

$$\nu = \frac{1}{2} \frac{R}{l} \frac{j(R)}{j_D}, \quad (15)$$

The complete solution of Eq. (14) has the form [11]

$$P(r) = \bar{P} + \left( \frac{r}{l} \right)^\nu \left[ C_1 I_\nu \left( \frac{r}{l} \right) + C_2 K_\nu \left( \frac{r}{l} \right) \right], \quad (16)$$

where  $I_\nu$ ,  $K_\nu$  are the modified Bessel functions of the first and second kind, respectively,  $C_1$ ,  $C_2$  are the integration constants.

It follows from  $P(\infty) = \bar{P}$  boundary condition, that  $C_1 = 0$ . The  $C_2$  constant can be found from the spin current continuity condition at the boundary between the probe and the film

$$\begin{aligned} J(R) &\equiv \frac{\hbar}{2e} \left\{ j(R)P(R) - j_D l \frac{dP}{dr} \Big|_{r=R} \right\} = \\ &= \frac{\hbar}{2e} Q_1 j(R) (\hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R)), \end{aligned} \quad (17)$$

where  $Q = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$  is the the probe spin polarization ( $\sigma_\pm$  being the partial conductivities in the probe),  $\hat{\mathbf{M}}(R)$  is the unit vector along the film magnetization near the probe,  $\hat{\mathbf{M}}_1$  is the same quantity for the probe. To obtain inverse population of the spin subbands in the film,  $(\hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R)) < 0$  condition is necessary.

The substitution of Eq. (16) into Eq. (17) gives

$$C_2 = \left\{ Q_1 (\hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R)) - \bar{P} \right\} \frac{j(R)}{j_D} \left( \frac{R}{l} \right)^\nu \frac{1}{K_{\nu+1} \left( \frac{R}{l} \right)}, \quad (18)$$

so that the electron spin polarization in the film near the probe takes the form

$$P(r) = \bar{P} + \left\{ Q_1 (\hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R)) - \bar{P} \right\} \frac{j(R)}{j_D} \left( \frac{r}{l} \right)^\nu \frac{K_\nu \left( \frac{r}{l} \right)}{K_{\nu+1} \left( \frac{R}{l} \right)}. \quad (19)$$

It follows from Eq. (19) that the spin polarization approaches monotonously to the equilibrium value with increasing the distance from the probe

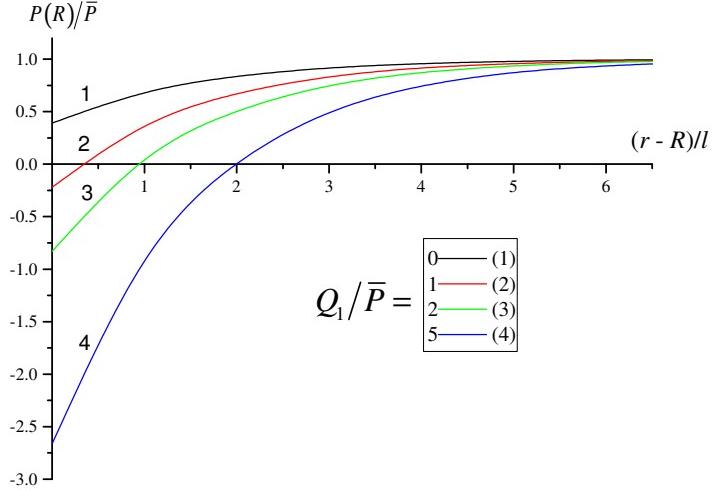


Figure 1: Spatial spin polarization distribution near the probe at  $R/l = 20$ ,  $j(R)/j_D = 1$  and various values of  $Q_1/\bar{P}$  ratio.

(Fig. 1). The maximal negative value of the nonequilibrium spin polarization  $\Delta P$  is reached at the probe boundary,

$$\Delta P(R) = \left\{ Q_1 \left( \hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R) \right) - \bar{P} \right\} \frac{j(R)}{j_D} \frac{K_\nu \left( \frac{R}{l} \right)}{K_{\nu+1} \left( \frac{R}{l} \right)}. \quad (20)$$

As to the dependence of the nonequilibrium spin polarization on the current density, it is necessary to have in mind that the current density  $j$  appears in Eqs. (19) and (20) not only as an explicit factor, but also in  $\nu$  parameter (see definition (15)). Because of such a reason, the spin polarization near the probe tends to a limiting value,  $P(R) \rightarrow Q_1 \left( \hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R) \right)$ . Note, that the nonequilibrium spin polarization appears in the case of a nonmagnetic probe also ( $Q_1 = 0$ ), that means replacing the spin-polarized electrons with non-polarized ones in the vicinity of the probe. In that case, the spin polarization remains positive and tends to zero at high current density ( $\Delta P \rightarrow -\bar{P}$ ,  $P \rightarrow 0$ ).

In Fig. 1, a coordinate dependence is shown of the spin polarization (referred to its equilibrium value) at given values of  $j(R)/j_D$  and  $R/l$  ratios and various values of  $Q_1/\bar{P}$  ratio. The spin polarization near the probe becomes negative at high enough values of the latter ratio.

In Fig. 2, the spin polarization on the probe boundary is shown as a function of the current density at given value of  $R/l$  ratio and various values of  $Q_1/\bar{P}$  ratio.

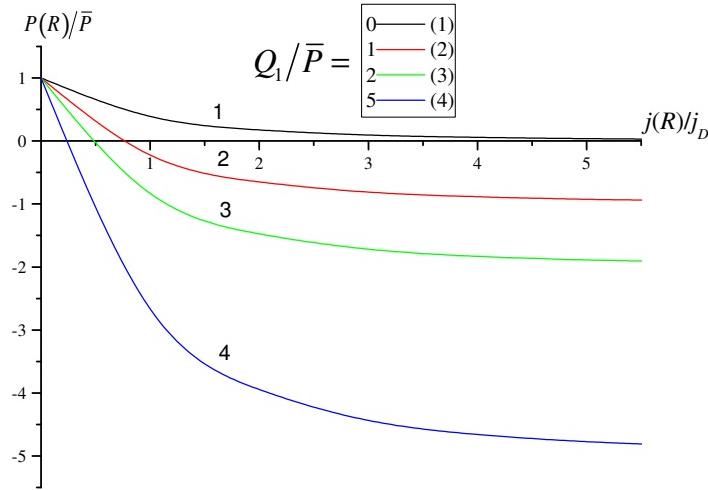


Figure 2: Spin polarization at the boundary between the probe and the film as a function of the (dimensionless) current density  $j(R)/j_D$  at  $R/l = 20$  and various values of  $Q_1/\bar{P}$  ratio.

As numerical analysis shows, the results weakly depend on  $R/l$  ratio. The parameters which influence the spin injection substantially are  $j(R)/j_D$  and  $Q_1/\bar{P}$  ratios.

At  $n \sim 10^{22} \text{ cm}^{-3}$ ,  $l \sim 10^{-6} \text{ cm}$ ,  $\tau \sim 10^{-12} \text{ s}$ , we have  $j_D \sim 10^9 \text{ A/cm}^2$ . With the current density in the probe  $j_0 \sim 10^8 \text{ A/cm}^2$  (such a value was reached, e.g., in [12]) and  $R/2h \geq 10$  (this ratio was equal to 5 in [12]), we find  $j(R) \sim 10^9 \text{ A/cm}^2 \sim j_D$ , that is sufficient, in accordance with Fig. 2, for negative spin polarization near the probe at  $Q_1/\bar{P} \geq 1$ .

## 4 Current-induced magnetic field and sd exchange effective field effects

At high current densities, influence becomes substantial of the current-induced magnetic field (the Ampere field). For rough estimates, we may use a simple formula for the magnetic field of a straight current

$$H = \frac{I}{R} \quad (21)$$

without factor 2 of current (“the half of a long wire”). The substitution of  $I = 2\pi Rhj(R)$  into Eq. (21) gives the magnetic field at the probe circumference with given current density:

$$H = 2\pi h j(R). \quad (22)$$

At  $h = 10$  nm and  $j(R) = 10^9$  A/cm<sup>2</sup> we have  $H \approx 600$  Oe.

If the film anisotropy field  $H_a$  is lower than the latter value, then the lattice magnetization near the probe is directed along the probe circumference. To align the probe magnetization opposite to the film magnetization, the probe anisotropy field  $H_{a1}$  is to be larger than the indicated  $H$  value, and the probe is to be magnetized by such a current of opposite direction that induces  $H > H_{a1}$  field at the probe circumference.

The inverse population is prevented also with the *sd* exchange interaction of the injected electrons with the magnetic lattice of the film, which tends to align the electron spins in the film parallel to ones in the injector.

The *sd* exchange energy is

$$U_{sd} = -\alpha \int (\mathbf{M}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r})) d^3\mathbf{r}, \quad (23)$$

where  $\alpha$  is the dimensionless constant of the *sd* exchange interaction,  $\mathbf{M} = M\hat{\mathbf{M}}$  is the lattice magnetization,

$$\mathbf{m} = \mu_B n P(\mathbf{r}) \hat{\mathbf{M}} \quad (24)$$

is the electron magnetization in the film,  $\mu_B$  is the Bohr magneton.

Substitution of Eqs. (24) and (19) into Eq. (23) gives the following expression for the nonequilibrium part of the *sd* exchange interaction energy:

$$\begin{aligned} \Delta U_{sd} &= -\alpha \mu_B n M \cdot 2\pi h \int_R^\infty \{P(r) - \bar{P}\} r dr = \\ &= -\alpha \mu_B n M \{Q_1 (\bar{\mathbf{M}}_1 \cdot \bar{\mathbf{M}}) - \bar{P}\} \frac{j(R)}{j_D} \cdot 2\pi R h l. \end{aligned} \quad (25)$$

The corresponding *sd* exchange effective field

$$\mathbf{H}_{sd} = -\frac{\delta \Delta U_{sd}}{\delta \mathbf{M}} \quad (26)$$

has an order of magnitude

$$H_{sd} \sim \mu_B \alpha n Q_1 \frac{j(R)}{j_D}. \quad (27)$$

This field becomes comparable with the molecular field  $\sim 10^6$  Oe at  $j(R) \sim j_D$ . To prevent switching antiparallel initial configuration ( $(\hat{\mathbf{M}}_1 \cdot \hat{\mathbf{M}}(R)) < 0$ ) to parallel one, it is necessary to pin the magnetization of the film by means of induced anisotropy with the aid of an antiferromagnetic substrate.

## 5 Conclusion

The analysis within a simplified model shows general possibility of reaching nonequilibrium negative spin polarization (the spin subband inversion) with using probe/film structures. However, realization of this possibility needs some action to prevent switching the antiparallel configuration to parallel one.

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